INDIAN STATISTICAL INSTITUTE
BANGALORE CENTRE
B. MATH III YEAR, II SEMESTER
MIDSEMESTER EXAMINATION
ANALYSIS IV

Max. Marks: 100 Duration: 3hrs

Questions 1,4,6 and 7 carry10 marks each; questions 2,3,5 and 8 carry 15 marks each.

1. Give an example of a set $A \subset R$ such that A is a set of first category in R but it is of second category when viewed as a metric space in its own right (with the restriction of the usual metric on R).

2. Let $X = (0,1) \cup (2,3)$ and d be the usual metric on X. Prove that there exists a metric d' on X such that d' is equivalent to d and (X, d') is complete.

3. If $\{T_n\}$ is a sequence of continuous linear maps between Banach spaces B_1 and B_2 such that $T(x) = \lim T_n(x)$ exists for all $x \in B_1$ show that T is a continuous linear map.

4. Show that there exist Lebesgue measurable functions $\{f_t : 0 \leq t \leq 1\}$ such that $\sup_{0 \leq t \leq 1} f_t$ is not Lebesgue measurable. Hint: Consider indicators of singleton sets.

5. If $f_n \to f$ in measure and $\{f_n\}$ is dominated by an integrable function then $f_n \to f$ in L^1.

6. Let $f, f_1, f_2, \ldots$ are measurable functions such that $f_1 \leq f_2 \leq \ldots$, $\int f_1^- dm < \infty$ and $\lim_{n \to \infty} f_n(x) = f(x)$ for all x. Show that $\int f_n dm$ and $\int f dm$ exist and $\lim_{n \to \infty} \int f_n dm = \int f dm$.

7. Let f be a continuous function on $[0,1]$. Prove that $\lim_{n \to \infty} (\int |f|^n dm)^{1/n} = \sup_{0 \leq x \leq 1} |f(x)|$. Hint: consider integral of $|f|^n$ over the set $E = \{x \in [0,1] : |f(x)| > M - \varepsilon\}$ where $M = \sup_{0 \leq x \leq 1} |f(x)|$.

8. If $(a,b] \subset \bigcup_{n=1}^{\infty} (a_n, b_n]$ prove that $b - a \leq \sum_{n=1}^{\infty} (b_n - a_n)$.

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PART B

IV Using SQL statements perform 15

1. Create Table emp with following description

Empno	-	Employee Number
Ename	-	Employee Name
Job	-	Job of Employee
Mgr	-	Employee Manager's Number
Hiredate	-	Date of Journey
Salary	-	Salary of Employee
Comm	-	Commission employee is getting
Deptno	-	Department employee belongs to

2. Display the structure of Table emp 1
3. Insert new row of data to emp Table 2
4. Display all rows of data which belong to deptno =100 2
5. Display all employees who earn salary more than 10000 2
6. Display employees whose job is system Analyst 2
7. Display employees whose commission is between 2000 and 5000 2
8. Increment salary by 15 % and commission by 5 % of salary 2

V Write a note on 5 x 3 = 15

1. Grant & revoke statements
2. Views and Indexes in SQL
3. Group functions in SQL 5 x 3 = 15

VI With Examples explain

1. Data Definition language in SQL
2. Data Manipulation Language 5

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